The Resting Membrane Potential

-70 mV
Voltmeter
\[ V = 0 \]

1 mM KCl  1 mM KCl

Permeable to K⁺

No net flux of K⁺
Initially $V = 0$

10 mM KCl  1 mM KCl

Net flux of $K^+$ from 1 to 2
Initial conditions $\rightarrow$ At equilibrium

Initially

$V = 0$

$V_{1-2} = -62 \text{ mV}$

10 mM KCl  1 mM KCl

10 mM KCl  1 mM KCl

Net flux of $K^+$ from 1 to 2

Flux of $K^+$ from 1 to 2 balanced by opposing membrane potential
Ion channels impart conductance (G)
\[ I_K = g_K(V_m - E_K) \]
How many ion channels make up for a given ionic conductance in a membrane?

A single channel can be represented as a conductor or a resistor.

\[ r_K = \frac{1}{\gamma_K} \]

\[ \gamma_K : \text{conductance of a single } K^+ \text{ channel} \]

\[ i: \text{current through a single channel} \]

Ohms law: \( V = RI \)

For a single channel: \( i = (V_m - E_{ion})/r_{ion} = \gamma_{ion} (V_m - E_{ion}) \)

hence, \( \gamma \) is the slope of the “I/V” plot.

\( V_m - E_{ion}: \text{Driving Force} \)

The total conductance of all \( K^+ \) channels in a cell:

\[ G_K = N_K \gamma_K \]

\( N_K: \text{total number of } K^+ \text{ channels} \)

The total \( K^+ \) current in a cell: \( I_K = G_K (V_m - E_K) \)
Important Ions in Neurons

Inside
Na$^+$ (5-15 mM)
K$^+$ (140 mM)
Cl$^-$ (4 mM)
Ca$^{2+}$ (0.1 μM)
A$^-$ (147 mM)

Outside
Na$^+$ (145 mM)
K$^+$ (5 mM)
Cl$^-$ (110 mM)
Ca$^{2+}$ (2.5-5 mM)
A$^-$ (25 mM)

**Equilibrium Potentials**

<table>
<thead>
<tr>
<th>Ion</th>
<th>Formula</th>
<th>Potential (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Na$^+$</td>
<td>$62 \log \frac{145}{5}$</td>
<td>90 mV</td>
</tr>
<tr>
<td></td>
<td>$62 \log \frac{145}{15}$</td>
<td>61 mV</td>
</tr>
<tr>
<td>K$^+$</td>
<td>$62 \log \frac{5}{140}$</td>
<td>-90 mV</td>
</tr>
<tr>
<td>Cl$^-$</td>
<td>$-62 \log \frac{110}{4}$</td>
<td>-89 mV</td>
</tr>
<tr>
<td>Ca$^{2+}$</td>
<td>$31 \log \frac{2.5}{10^{-4}}$</td>
<td>136 mV</td>
</tr>
<tr>
<td></td>
<td>$31 \log \frac{5}{10^{-4}}$</td>
<td>146 mV</td>
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</tbody>
</table>

Figure 2.1: Ion concentrations and Nernst equilibrium potentials (2.1) in a typical mammalian neuron (modified from Johnston and Wu 1995). A$^-$ are membrane-impermeant anions. Temperature $T = 37^\circ C$ (310$^\circ K$).
\[ V_m = -65 \text{ mV} \]
\[ E_K = -90 \text{ mV} \]
\[ E_{Cl} = -89 \text{ mV} \]
\[ E_{Na} = +61 \text{ mV} \]

\[ I_{ion} = G_{ion}(V_m - E_{ion}) \]

Ion movements in terms of currents

- **Inward current**: Positive Charge entering the cell (Cations in or Anion out)
- **Outward current**: Positive Charge exiting the cell (Cations out or Anions in)

By convention, positive current is outward.
Figure 2.4: Mechanistic interpretation of the resting membrane potential (2.4) as the center of mass. Na\(^+\) conductance increases during the action potential.
From Hodgkin & Katz 1949
The phospholipid bilayer imparts capacitance C

The ability to store charge Q
C=Q/V

Q=CV
I = dQ/dt=C*dV/dt
\[ I_m = I_c + I_i \]
\[ I_i = I_{Na} + I_K + I_{Cl} \]
\[ I_c = C_m \frac{dV_m}{dt} \]

\[ I_m = C_m \frac{dV_m}{dt} + g_K(V_m - E_K) + g_{Na}(V_m - E_{Na}) + g_{Cl}(V_m - E_{Cl}) \]
The membrane time constant $\tau$

$$V_m(t) = \frac{1}{g_m} I_m (1 - e^{-t/\tau})$$

$$V_m(t) = \frac{1}{g_m} I_m e^{-t/\tau}$$

$$g_m = g_K + g_{Na} + g_{Cl}$$

$$V_{\infty} = \frac{1}{g_m} I_m$$

$$R_m = \frac{1}{g_m}$$

$$\tau = R_m C_m$$
The Sodium Potassium Pump Keeps the Gradient Intact